NOTE.—I have used capitals A, B, C, R, S, and T in some cases where small letters are used in the problem as published in the May MONTHLY. This substitution of capitals for small letters is done for the sake of clearness in distinguishing vectors from their tensors.

2708 [May, 1918]. Proposed by WILLIAM HOOVER, Columbus, Ohio.

A uniform plank of length $2a$ and thickness $2h$ rests in equilibrium on a fixed rough horizontal cylinder of radius c, so that a vertical plane containing the dimension $2a$ and the center of gravity of the plank is at right angles to the axis of the cylinder; find the period of a complete small oscillation of the plank.

SOLUTION BY THE PROPOSER.

Let G be the center of gravity of the plank; G_0 the initial position of G; O the center of gravity of the cylinder and vertically beneath G_0; φ = the angular rotation of the plank after any time t from the beginning of motion, OX, OY the horizontal and vertical coördinate axes $OA = x$, $GA = y$, the coördinates of G, $k = \sqrt{(a^2 + h^2)/3}$ = the radius of gyration of the plank about an axis parallel to the axis of the cylinder. We obtain

$$x = (c + h) \sin \varphi - c\varphi \cdot \cos \varphi, \tag{1}$$

$$y = (c + h) \cos \varphi + c\varphi \cdot \sin \varphi. \tag{2}$$

By *vis viva*,

$$\tfrac{1}{2}m(\dot{x}^2 + \dot{y}^2 + k^2\dot{\varphi}^2) = - mgy + C. \tag{3}$$

From (1),

$$\dot{x} = (h \cos \varphi + c\varphi \sin \varphi)\dot{\varphi}, \tag{4}$$

$$\dot{y} = (- h \sin \varphi + c\varphi \cos \varphi)\dot{\varphi}; \tag{5}$$

(3) then is

$$\frac{m}{2}(h^2 + k^2 + c^2\varphi^2)\dot{\varphi}^2 = C - mg\{(c + h) \cos \varphi + c\varphi \sin \varphi\}. \tag{6}$$

Differentiating both sides of (6) with respect to t using the value of k,

$$3c^2\varphi \cdot \dot{\varphi}^2 + \{(a^2 + 4h^2) + c^2\varphi^2\}\ddot{\varphi} = - 3g\{- h \sin \varphi + c\varphi \cos \varphi\}. \tag{7}$$

Let φ be so small that we may put $\sin \varphi \doteq \varphi$, $\cos \varphi = 1$, and omit φ^2, $\dot{\varphi}^2$, etc., because of the nature of the oscillation; we have, then,

$$(a^2 + 4h^2)\ddot{\varphi} = - 3g(c - h) \cdot \varphi, \tag{8}$$

an harmonic equation in φ if $c > h$, giving the period required,

$$T = 2\pi\sqrt{\frac{a^2 + 4h^2}{3g(c - h)}}. \tag{9}$$

It may be instructive to derive (8) by another method given by Holditch in the eighth volume of the *Cambridge Transactions* and quoted by Routh, *Dynamics of a System of Rigid Bodies, Elementary Part*, fourth edition, 1882, pages 341–342.

Let the motion of a body in space of two dimensions be given by the coördinates x, y of its center of gravity, and the angle φ which any fixed line in the body makes with a line fixed in space; α = the equilibrium value of φ; x', x'', etc., denoting $dx/d\varphi$, etc.; x_0', etc., the values of x', etc., when $\varphi = \alpha$, and k = the principal radius of gyration; then

$$(x_0'^2 + k^2)\ddot{\varphi} = - gy_0''\varphi. \tag{10}$$

From (4) and (5) we have, with $\varphi_0 = 0$,

$$x_0' = h, \qquad y_0'' = c - h \tag{11}$$

(11) in (10) gives (8).

Also solved by R. C. COLWELL and J. B. REYNOLDS.

2711 [June, 1918]. Proposed by PAUL CAPRON, U. S. Naval Academy.

Show that the curves (*a*) $a^8y_1^2 = x^4(a^2 - x^2)^3$, (*b*) $a^8y_2^2 = x^8(a^2 - x^2)$ bound ten areas, of which two are each $(a^2/4)(\frac{1}{4}\pi - \frac{1}{3})$ and the remaining eight are each $a^2/24$.